3008

B. Tech. 1st Semester (CSE) Examination – February, 2022

MATH - I (Calculus and LINEAR ALGEBRA)

Paper : BSC-MATH-103-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Prove that $\beta(m, n) = \beta(n, m)$.

(b) Compute $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & d \end{bmatrix}$.

(c) Find $|A|$ where the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

(d) Let $T : U \to V$ be a linear transformation the prove $T(0_u) = 0_v$ where 0_u and 0_v are zeros vectors in U and V respectively.

(e) Let $T_1 : R^2 \to R^2$ and $T_2 : R^2 \to R^2$ be linear transforms, then verify that $T_1 T_2$ and $T_2 T_1$ are well defined or not.

(f) Define inner products space.

UNIT – I

2. (a) Examine for extreme values of :

$$f(x, y) = 3x^2 - y^2 + x^3.$$

(b) Show that :

$$\log(x + h) = \log x + \frac{h}{x} - \frac{h^2}{2x^2} + \dots\dots\dots + (-1)^{n-1} \frac{h^n}{n(x + \theta h)^n}$$

3. (a) Find the volume of solid formed by revolution about x-axis of loop of the curve $y = \left\{ \frac{ax^2 + x^2}{(a - x)} \right\}^{1/2}$.

(b) Show that $\beta(m, n) = \int_0^\infty \frac{y^{n-1}}{(1 + y)^{m+n}} dy$

UNIT – II

4. (a) Solve following equations with the help of matrices :

$x + y + z = 3,\ x + 2y + 3z = 4,\ x + 4y + 9z = 6$

(b) Show that vectors $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$ and $x_4 = (-3, 7, 2)$ are C.D. and find relation between them.

5. (a) Using Gauss-Jordan method find inverse of matrix :

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

(b) Solve the following equations by Cramer's rule $6x + y - 3z = 5, x + 3y - 2z = 5, 2x + y + 4z = 8$.

UNIT – III

6. (a) If $P(x)$ denotes the set of all polynomials in x over a field F_1 then show that $P(x)$ is a vector space over F with vector addition as polynomial's addition and scalar multiplication defined as product of polynomial by an element of F.

(b) Examine the linear independence of the following vectors :

$(1, 1, 1), (1, 2, 3), (0, 1, 2)$ in R^3.

7. (a) Verify that mapping $T : R^3 \to R^2$ defined by $T(x, y, z) = (2x - 3y, 7y + 2z)$ is linear transformation.

(b) Find a linear transformation $T : R^3 \to R^3$ where range space is spanned by vectors $(1, 2, 3), (4, 5, 6)$.

UNIT – IV

8. (a) Find eigen values and eigen vector of :

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Show that matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not digonalizable over the field C.

9. Prove that every finite dimensional inner product space has an orthogonal basis.