

97751

BCA 1st Semester 4 Year Course

Examination, December-2024

MATHEMATICS

Paper-23BCA401DS01

Mathematical Foundation of Computer Science

Time allowed : 3 hours]

[Maximum marks : 70

Note : Students have to attempt **five** questions in total, **first** being compulsory and selecting **one** from each unit.
All questions carry equal marks.

1. (a) How many number of subset of a set having n elements.
- (b) Define equivalence relation ?
- (c) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
- (d) Discuss the removable discontinuity.
- (e) Find $\frac{dy}{dx}$ for $y = \sin^{-1}(x)$.
- (f) If $\tan A = \frac{4}{3}$ then evaluate $\cos A$ and $\sin A$
- (g) Evaluate the value of $\int_0^1 \frac{dx}{1+x^2}$.

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[P.T.O.]

Unit-I

2. (a) If $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$ then find
- (i) $A \cup B$
 - (ii) $A - B$
 - (iii) $A \cap B$
- (b) In a group of 65-people, 40 like cricket, 10 like both cricket and tennis. find:
- (i) How many like tennis?
 - (ii) How many like tennis only and not cricket.
3. (a) If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = \frac{5x+3}{7} : x \in \mathbb{R}$.
Prove that f is bijective function and hence find the inverse of f .
- (b) If the map $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = \log(1+x)$ and the map $g: \mathbb{R} \rightarrow \mathbb{R}$ is given by $g(x) = e^x$ find $(g \circ f)x$ and $(f \circ g)x$.

Unit-II

4. (a) If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, find $\tan(2A+B)$.

- (b) Show that $\lim_{x \rightarrow 0} \frac{e^{\sqrt{x}} - 1}{e^{\sqrt{x}} + 1}$ does not exist.

5. (a) Test continuity of the function

$$f(x) = \begin{cases} (x-a) \sin \frac{1}{x-a} & : x \neq a \\ 0 & : x = a \end{cases}$$

- (b) Prove that

$$3 \cos^2 \frac{\pi}{4} + \sec \frac{2\pi}{3} + 5 \tan^2 \frac{\pi}{3} = \frac{29}{2}$$

Unit-III

6. Differentiate the following function with respect to x.

(a) $y = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$

(b) $y = \left(\frac{\sin x + e^x}{1 + \log x} \right)$

7. Differentiate the function with respect to x.

(i) $y = \frac{\sqrt{1+\cos x}}{\sqrt{1-\cos x}}$

(ii) $y = x^{\log x}$

Unit-IV

8. (a) Let $f(x) = x^2 - 5x + 6$, find $f(A)$, If

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

- (b) Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(a+b+c)$

9. (a) Solve the given system of linear equations by matrix method.

$$x - y - z = 1, 2x + y + z = 2, x - 2y + z = 4$$

(b) If $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 7 \\ 0 & -5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & -4 \\ -6 & 8 & -8 \end{bmatrix}$

Prove that $(AB)^t = B^t A^t$.